

# Modeling Companion E

## Poverty Trap Models

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### LEARNING OBJECTIVES

- What do poverty traps consist of?
- How do we model a poverty trap by extending the Solow model?
- Why and how can foreign aid be useful in the case of poverty traps?

### MOTIVATION

In chapters 4 and 5, we discussed how poor countries and poor households can face great difficulties in getting on to the first rung of the development ladder. One reason is that at very low levels of income, households and countries are often facing poverty traps.

In this modeling companion, we will clarify what we mean by "poverty traps," and we will study three specific examples. We will then extend the Solow model to show how modifications of some of the assumptions can drastically change the dynamics of the Solow model and lead to very different predictions and implications for the role of foreign aid.

To delve further on this topic, read through the list of further reading provided at the end of this companion.

### INTRODUCTION

Poverty traps emerge from situations with multiple equilibria. In the context of economic growth, one equilibrium is often characterized by a very low level of income per capita (thereby termed the "bad" equilibrium) while another equilibrium is characterized by a high level of income per capita (the "good" equilibrium). Because of "self-enforcing" or "attractive" dynamics, once the system is in one equilibrium (say, the bad one), it is impossible for the system to reach the other equilibrium.

We will study three specific examples: the industrialization trap, the savings trap, and the demographic trap.

### EXAMPLES OF POVERTY TRAPS

#### **Example 1: the Industrialization Trap**

The diagram below illustrates the two self-enforcing, "attractive" dynamics that would take place in an industrialization trap. On the left hand side is featured the "vicious" cycle: an economy starting with low levels of manufacturing generates little income to invest in infrastructure, human capital, and technology. And conversely, poor infrastructure coupled with low levels of skills and technologies are not conducive to supporting a vibrant manufacturing sector. On the opposite hand,

once a modernized industry exists, it generates enough revenue such that some is reinvested to develop infrastructure, human capital, and technologies, which will in turn contribute to strengthening the industrial sector.



Is it possible to change the vicious cycle into a virtuous one? Through large investments in infrastructure, skills, and R&D, it might be possible to change the vicious dynamics. This is a simple and compelling rationale for aid. In that model, aid would act as a one-time "big push," pushing the economy away from the vicious cycle.

### **Example 2: The Saving Trap**

Households that earn very low income are usually unable to save any money because almost all income is spent on consumption. Without savings, it is impossible to invest in education or in some equipment to start a small business, which would in turn provide opportunities to earn a higher wage. As a result, we obtain a vicious cycle where poverty causes low savings, which once again leads to deeper poverty. On the opposite hand, there exists a virtuous cycle, where high-income households can save enough to invest in critical areas such as education, health, or equipment for a small business. This in turn guarantees a greater likelihood to earn a higher income. Microcredit could be one way to transform the vicious cycle into a virtuous one by enabling poor households to realize critical investments.



### **Example 3: The Demographic Trap**

Poor households often have strong incentives to have many children (due to high child mortality rates, need for farm labor, etc.). In turn, a high fertility rate makes it difficult for households to invest the necessary resources for developing the potential of their children (education, health etc.), which give children little chance to get out of poverty. Hence a vicious cycle is born. On the opposite hand, when the fertility rate is low, families are able to invest the necessary resources in their children, guaranteeing them greater opportunities to earn higher wages. As they get richer, they

have fewer incentives to have many children, and a virtuous cycle perpetuates. Family planning and free education are possible ways to transform the vicious cycle into a virtuous one.



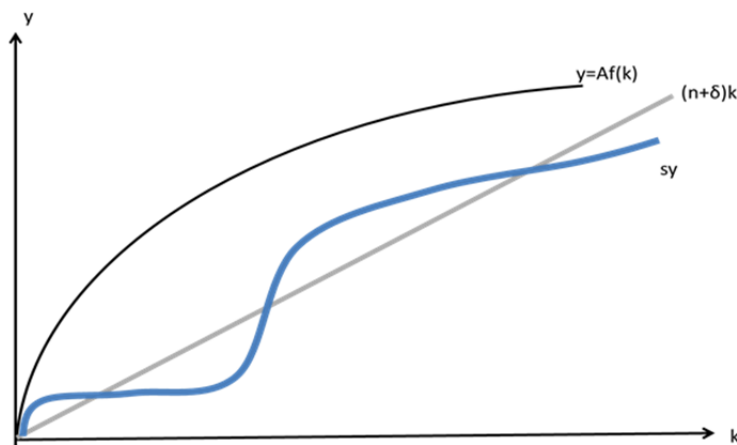
## POVERTY TRAPS MODELS

We will now build on the Solow model and show how poverty traps can be modeled. In the Solow model, parameters such as the population growth ( $n$ ), the saving rate ( $s$ ), or the level of technology ( $A$ ) are taken as exogenous constants. As we examine different cases of poverty traps, we will relax these assumptions.

### 1. The Savings Trap

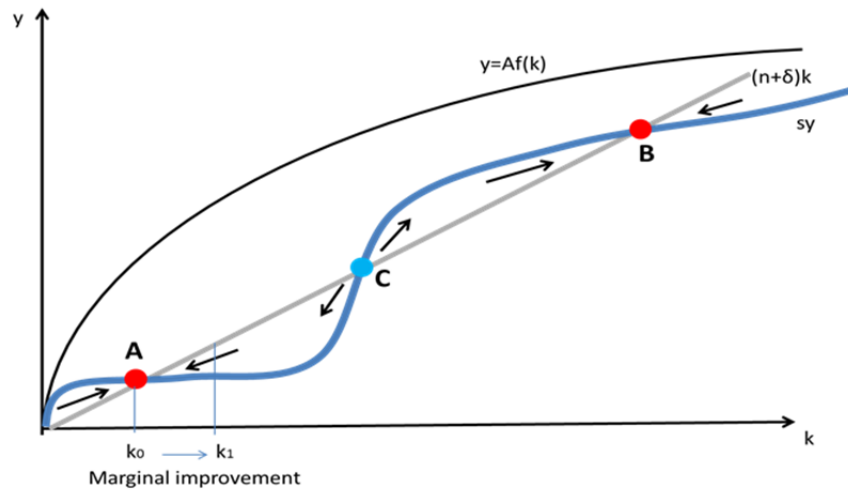
In the Solow model, the saving rate is an exogenous constant. Let's now model more precisely how the saving rate might change as income per capita increases. In particular, let's assume that as income per capita increases, the saving rate increases. In other words, let's build into the model the empirical fact that poor people are not able to save as much as rich people are.

The graph below illustrates the key variables. The main difference from the standard Solow model is that the saving curve now displays an "S-shape." The other curves remain unchanged.

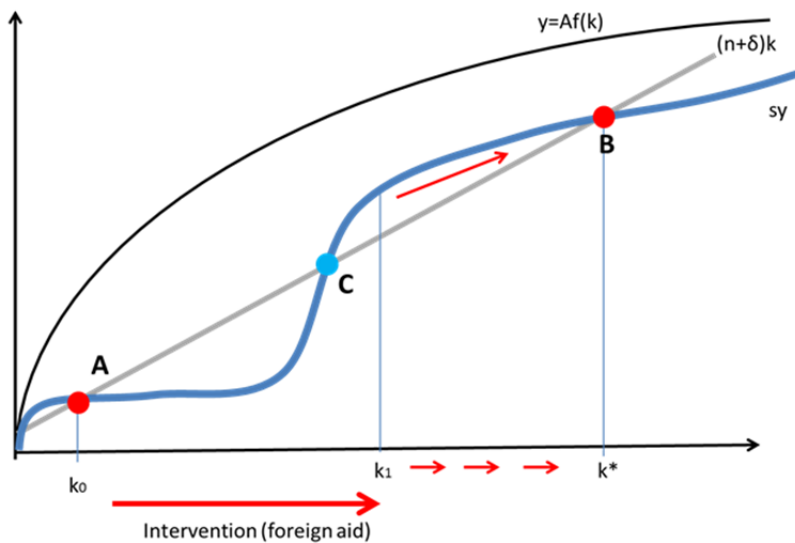


We note that the blue S-shaped curve crosses the  $(n + \delta)k$  line three times. As noted on the graph below, we now have three equilibria in this model: A, B, and C. Only A and B are stable equilibria, while C is an unstable equilibrium. The key feature of a stable equilibrium is that it is attractive: in other words, a marginal improvement (for example moving from  $k_0$  to  $k_1$ ) sends you back (from  $k_1$ ,

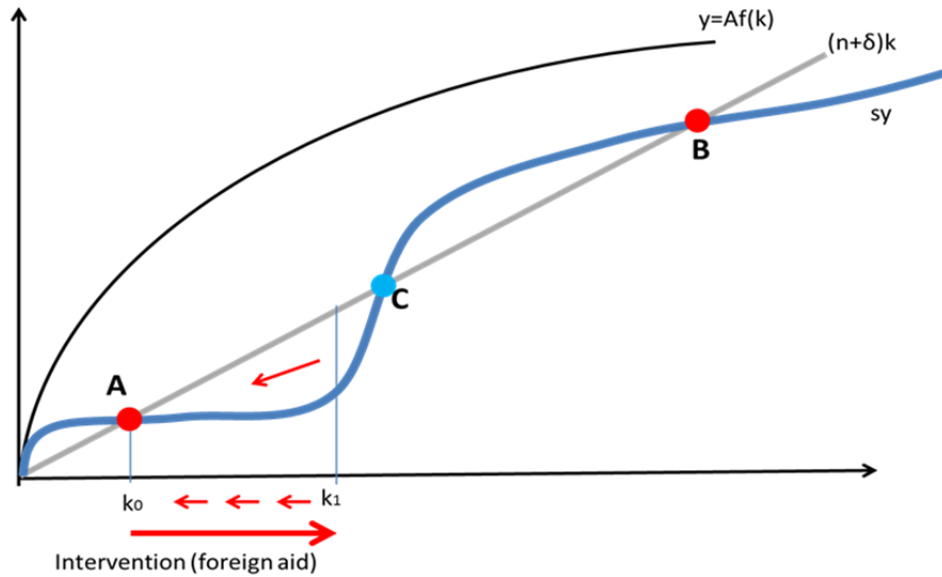
we would move back to  $k_0$ ). As a result, depending on the initial condition, we might converge to one or the other stable equilibria: "rich countries might be in B while poor countries are stuck in A."



Can external interventions, such as foreign aid, be helpful when there are multiple equilibria? Suppose the economy is at A, where capital per worker equals  $k_0$ . Suppose that the economy receives a transfer that brings its level of capital per worker to  $k_1$ . If  $k_1$  is above C, then the attractive dynamics of equilibrium B will take place, and the economy will converge toward the high equilibrium, B.



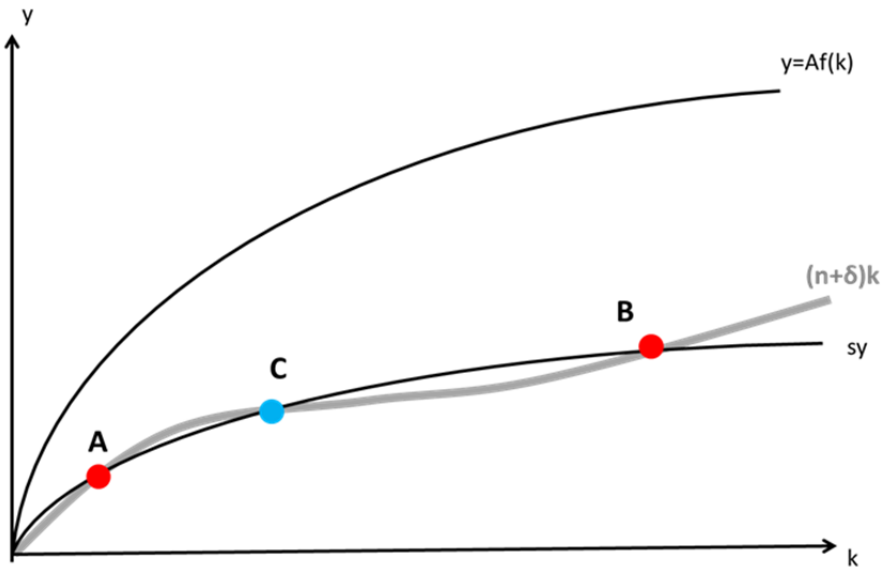
However, as shown on the graph below, if  $k_1$  is below C, then the attractive dynamics of equilibrium A will take place, and the economy will converge toward the low equilibrium, A.



The conclusion is that foreign aid can help if and only if the one-time transfer is high enough to get past point C.

## 2. The Demographic Trap

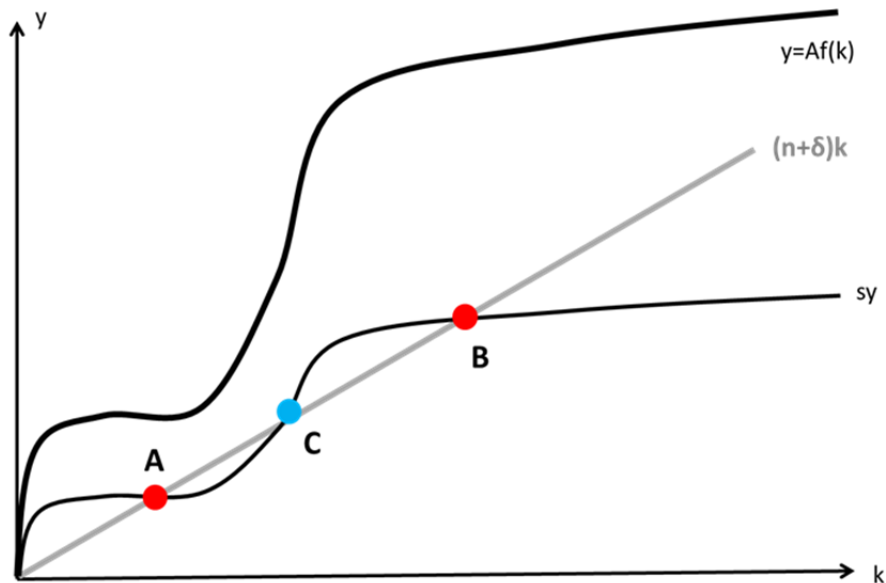
Let's now assume that the saving rate is constant, and we will relax the assumption on the population growth rate. As we have seen, as people get richer, they tend to have fewer children. The graph below represents a demographic transition as such. We see that the line  $(n+\delta)k$  is steeper for low levels of capital, and flatter for higher levels of capita. This is because we now assume that the population growth rate decreases as capital per worker and income per capita increases.



Similar to the savings trap, we have three equilibria, two of them being stable. The dynamics and implications for the role of foreign aid are therefore similar to the savings trap.

### 3. The Industrialization Trap

Let's now assume that the population growth rate is constant, and we will relax the assumption on the technology level. Let's assume that as the economy develops, its overall level of technology also increases. The graph below represents such an evolution. We see that the curve  $y = Af(k)$  is now S-shaped: this is because the level of technology  $A$  is low at low levels of capital per worker and then increases.



Similar to the savings and the demographic traps, we have three equilibria, two of them being stable. The dynamics and implications for the role of foreign aid are therefore similar to before.

## MODELING ACTIVITIES

Remember our Solow-type economy from modeling companion D:

- the economy produces output according to the production function:  $y_t = A k_t^\alpha$  (1), where  $0 < \alpha < 1$ ,  $y$  is the output per worker,  $A$  is the level of technology, and  $k$  is capital/worker.
- Capital accumulates according to the following 'Law of Motion':  $k_{t+1} = k_t + sy_t - nk_t$  (2), where  $k$  is capital per worker,  $s$  is the savings rate and  $n$  is the population growth rate.

We assume there is no depreciation, i.e.  $\delta = 0$ . The  $t$  subscript denotes the time period (e.g. if  $t = 2000$  then  $t+1 = 2001$ , and so on).

*Open the spreadsheet 'Modeling Companion E Poverty Trap Model.xls'*

## **A. The Demographic Trap**

In modeling companion D, we assumed that the population growth rate was constant. After studying the situation in low-income countries, you hypothesize that population growth rates are not a simple constant. When income is below a certain level (a poverty line), population growth rates are quite high, and when income exceeds that basic level, population growth rates are smaller. This can be expressed as follows:

$$n = 0.08 \quad \text{if } y_t < 2$$

$$n = 0.04 \quad \text{if } y_t \geq 2$$

This is called a demographic transition. You may think of  $y_t = 2$  as a poverty line.

1) Justify why you make this hypothesis. In other words, why do you think poorer households would have more children?

2) In tab "Demographic Trap 1" of the excel spreadsheet, we investigate the dynamics of savings and depreciation with and without a demographic transition, for values of  $k$  between 0 and \$15/person. You will see that we assume some values for the parameters  $s$ ,  $\alpha$ , and  $A$ .

a) In the simulation with a demographic transition (columns J and K), does the population growth rate change? If so, for what value of capital does it change?

Graph 1 displays  $sy$  and  $nk$  as a function of  $k$  when there is a demographic transition. Graph 2 displays  $sy$  and  $nk$  as a function of  $k$  when there is no demographic transition.

b) Looking at the graphs, can you say if there will be any stable or unstable steady states in the model with a demographic transition? If so, how many? How is it different from the case with no demographic transition?

3) In tab "Demographic Trap 2" of the excel spreadsheet, we simulate a Solow-type economy with a demographic transition and with initial condition  $k_0 = 1$ .

a) What is the population growth rate in this simulation? Is it always the same? Why?

b) Plot income per capita as a function of time. Describe and comment.

c) Change the initial condition and see what happens. Try for example  $k_0 = \{1; 3; 8; 9; 12; 25\}$ .

Plot on the same graph income per capita as a function of time for  $k_0 = 8$  and  $k_0 = 9$ .

d) Why is this a model of a poverty trap?

4) Suppose we start with  $k_0 = 8$ . What happens if in 2010, the economy receives an international aid transfer of \$2.50 in capital per worker. To help you answer this question, simulate the cash transfer in excel and look at the graph of  $y$  as a function of time.

5) Suppose  $k_0 = 1$ . Is the transfer of \$2.50 in capital per worker improving the economy in the long-term?

6) Building on the simulations from modeling companions D and E, explain why people might disagree about the long-term effects of foreign aid.

## **B. The Savings Trap**

Now suppose that the saving rate can be modeled by the following rule:

$$s = 0.02 \quad \text{if } y_t < 2$$

$$s = 0.25 \quad \text{if } y_t \geq 2$$

1) Can this rule realistically model what happens in the real world?

In tab "Savings Trap 1" of the excel spreadsheet, we investigate the dynamics of savings and depreciation with and without a savings trap, for values of  $k$  between 0 and \$15/person. You will see that we assume some values for the parameters  $n$ ,  $\alpha$ , and  $A$ .

2) Looking at the graphs, can you say if there will be any stable or unstable steady states in the model with a savings trap? If so, how many? How is it different from the case with no trap?

In tab "Savings Trap 2" of the excel spreadsheet, we simulate a Solow-type economy with a savings trap and with initial condition  $k_0 = 1$ .

3) Try out different initial conditions and comment on the results. Argue why this model is called a savings trap.

## **C. The Industrial Trap**

Now suppose that the production function can be modeled by  $y_t = A k_t^\alpha$  with:

$$A = 1.5 \quad \text{if } k_t < 6$$

$$A = 3.5 \quad \text{if } k_t \geq 6$$

where  $0 < \alpha < 1$ ,  $y$  is the output per worker,  $A$  is the level of technology, and  $k$  is capital per worker. Capital still accumulates as follows:  $k_{t+1} = k_t + sy_t - nk_t$ , where  $s$  is the saving rate and  $n$  is the population growth rate.

1) Explain under what circumstances an economy would have a production function as described above.

In tab "Industrial Trap 1" of the excel spreadsheet, we investigate the dynamics of savings and depreciation with and without an industrial trap, for values of  $k$  between 0 and \$15/person. You will see that we assume some values for the parameters  $s$ ,  $\alpha$ , and  $n$ .

2) Looking at the graphs, can you say if there will be any stable or unstable steady states in the model with an industrial trap? If so, how many? How is it different from the case with no trap?

In tab "Industrial Trap 2" of the excel spreadsheet, we simulate a Solow-type economy with an industrial trap and with initial condition  $k_0 = 1$ .

3) Try out different initial conditions and comment on the results. Argue why this model is called an industrial trap.



## **D. The Industrial Trap with Transfers**

We will now simulate what happens when an economy with a poverty trap receives external transfers such as foreign aid. In the tab "Industrial Trap with transfer," we have simulated the same Solow economy as in part C adding in transfers. We simulate a transfer in 2030 to the physical capital stock that is equivalent to \$2 per person and to \$4 per person.

- a) Describe what happens to income per worker and the growth rate of income in the short-term and in the long-term.
- b) Under what conditions can a transfer change any of the dynamics of the Solow model?
- c) From this simulation, what can you conclude about the role of foreign aid in a Solow-type economy with a poverty trap?

## **FURTHER READING**

This article reviews models of self-reinforcing mechanisms that cause poverty to persist.

[Azariadis, C. & Stachurski, J. \(2005\). Poverty traps. \*Handbook of Economic Growth\*, 1 \(A\).](#)

This article explores the useful distinction between chronic and transitory poverty in understanding rural welfare dynamics, highlighting the possibility of poverty traps.

[Barrett, C. B. \(2005\). Rural poverty dynamics: development policy implications. \*Agricultural Economics\*, 32 \(s1\).](#)

This paper offers an informal theory of "fractal" poverty traps: poverty traps with multiple dynamic equilibria existing simultaneously at multiple scales of analysis and self-reinforcing through feedback effects.

[Barrett, C. B. & Swallow, B. M. \(2006\). Fractal poverty traps. \*World Development\*, 34 \(1\).](#)

This paper explores the idea that simultaneous industrialization of many sectors of the economy can be profitable for them all even when no sector can break even industrializing alone.

[Murphy, K. M., Shleifer, A., & Vishny, R. W. \(1989\). Industrialization and the big push. \*Journal of Political Economy\*, 97 \(5\).](#)