Modeling Companion A
National Income Accounting

LEARNING OBJECTIVES

- What is an economy?
- How do we measure the size of an economy (GDP, GDP per capita, GDP PPP)?
- How do we define economic growth?
- The rule of 70
- The log scale

MOTIVATION

1.5 Shenzhen, China, in 1980

"Looking northwest . . .," Leroy W. Demery, Jr., Flickr. Used with permission.

1.6 Shenzhen, China, in 2002

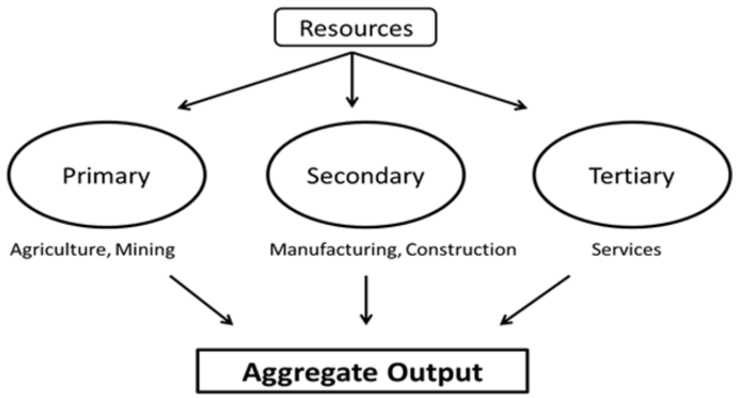
Reuters.

These two photos, displayed in Chapter 1 of ASD, feature the same geographic region: Shenzhen. In 1980, a rural village of about 30,000 people; today, a modern city of about 12 million. These photos look drastically different and when trying to explain what happened, one might want to invoke the concept of economic development. But what exactly is economic development? More fundamentally, what is an economy, and how does it develop? Didn't the rural village of Shenzhen have an economy as well in the 1980s? In what follows we will attempt to provide a few answers. Importantly, the key difference between Shenzhen in 1980 and Shenzhen today is the scale (as well as the nature) of its economy. Measuring the size of an economy is therefore going to be critical because it will enable us to track changes through time as well as compare it to other countries.

A. WHAT IS AN ECONOMY?

DEFINITION: An Economy An economy is a system of *resource allocation* for production, consumption, and investment within a given region (such as a country).

Below we schematically represent the various resources available in the economy being allocated to different production processes. In other words, resources are used as inputs to the production processes, and for that reason, we also call them factors of production.



What sorts of resources (or factors of production) are available in an economy? Here is a short list along with the usual notation that economists like to use.

- Labor (L): the amount of hours worked by workers.
- Physical Capital (K): plant, machinery, equipment, as well as infrastructure (bridge, roads, etc.).
- Land and Natural resources (R or Z): arable lands, minerals, fossil fuels, forests, etc.
- Human Capital (H): education, skills, efficiency of the worker, etc.
- Technology and Organization (A): inventions, patents, techniques, knowledge, systems of organization, institutions (rules and norms), culture, etc.

For example, take the farming sector. A hypothetical allocation of resources in the farming sector would consist of using labor and land to produce food. As such, we are modeling the production process associated with farming, and we stipulate that in order to produce food, we only need labor and land. That is why we write Y = F(Z,L), where F is the production function of the farming process. This might have been accurate at the very beginning of agriculture where very little machinery was used. A more appropriate production function to model modern agriculture would use not just labor and land as inputs, but also physical capital as well as knowledge, i.e. F(Z,L,K,A). In Modeling Companion E, we will introduce different types of production functions and dive more into their specific properties.

B. MEASURING THE SIZE OF AN ECONOMY

DEFINITION: GDP

GDP is a measure of aggregate production (or output) within a country during a given period of time, typically one year. In other words, it is the value of all the goods and services produced in the country.

DEFINITION: GNP

GNP is a measure of the income of a country, i.e. the value of all goods and services produced in one year by labor and property supplied by the residents of a country.

EXAMPLE

GDP or GNP?

Suppose the country is an oil exporter, and the government owns two-thirds of the oil, while foreign companies own one-third. The GDP would count all of the oil produced within the country, but national income would include only the two-thirds of the oil owned by the government. Hence, in that case, GNP would be less than GDP.

Efforts to consistently measure economic activities intensified after the Great Depression in the 1930s. Today international standards are defined by the United Nations System of National Accounts. There are three main ways of calculating GDP:

- Production: by adding the value of all the sales of goods and services in the country and subtracting the value of intermediate consumption (to avoid double-counting)
- Income: by using data on income received by the various agents of the economy, i.e. wages that workers receive in exchange for labor, interest on capital, rent for land, and profits for entrepreneurs.

$$GDP = Domestic\ Income = Wages + Interests + Rent + Profits$$

- Expenditures: by using data on total expenditure of money to buy goods and services from all the agents in the economy.

$$GDP = Domestic\ Expenditures = Private\ Consumption + Private\ Investment + Government\ Expenditures + Exports - Imports$$

Data used to calculate these come from many different sources: census data, tax data, customs, surveys, government budgets, etc. This is why GDP is updated several times from the advance quarterly estimates one month after the end of the quarter to five year benchmark revisions.

DEFINITION: GDP per Capita

GDP per Capita, or Per Capita Income, is the GDP of a country divided by the amount of people in the country: GDP per Capita = GDP / Population.

While GDP on its own is an important measure of an economy's overall size, it is also important to look at GDP per person because it provides an idea of what citizens' average incomes are, and hence gives an average measure of economic quality of life.

DEFINITION: PPP

Purchasing Power Parity is the amount of adjustment needed on the exchange rate between countries in order for the rate to be equivalent to each currency's purchasing power.

GDP provides a way of ranking economies in terms of pure size in units of currency. However, units of currency can also be difficult to compare between countries because the costs of living in those countries might be very different. A classic example of this is *The Economist* magazine's Big Mac index, which measures how much a McDonald's Big Max costs in dollars. As of 2013, the most expensive was in Switzerland, at $7.54, while the least expensive was in Russia at $1.36. The fact that an identical item costs five times as much in Switzerland than in Russia indicates that the same amount of currency has different values in each location. More generally, economists often adjust GDP figures by multiplying them by a Purchasing Power Parity (PPP) factor calculated by comparing local costs for comparable baskets of goods. PPP estimates more adequately capture the actual cost of living in a country than simple exchange rate measures.

EXAMPLE

	LUXEMBOURG	**SUDAN**
GDP	60 billion	67 billion
Population	0.54 million people	38.0 million people
GDP per Capita	~ 111,000 $/person	~ 1,800 $/person
GDP per Capita PPP	~ 91,000 $PPP/person	~ 3,400 $PPP/person

Data from World Bank Database, for the year 2013. Values in current international dollar.

DEFINITION: GDP at constant prices

GDP at constant prices is also called real GDP. It is an inflation-adjusted measure of GDP. Because inflation changes the value of money over time, we cannot directly compare values of current (or nominal) GDP in the present and in the past.

C. ECONOMIC GROWTH

DEFINITION: Rate of Growth

Rates of growth are defined in terms of percent change in a variable. We will often use γ (gamma) to signify a growth rate. In discrete time, we can approximate growth rates of a variable X by:

$\gamma_X = \frac{X_{t+1} - X_t}{X_t} = \frac{\Delta X_t}{X_t}$, where X_t is the value of the variable X at time t.

DEFINITION: Economic Growth

Economic growth measures the change in the GDP over a given period, for example, the current year relative to the past year.
Economic growth signifies an increase in GDP.

EXAMPLE

Below we list the levels of GDP per capita in China between 2010 and 2013.
We then accordingly calculate what the rates of growth have been in in 2011, 2012, and 2013.

We use the following formula, where $X = \frac{GDP}{Cap}$: $\gamma_{X_{2011}} = \frac{X_{2011} - X_{2010}}{X_{2010}}$

Year	GDP per capita, PPP (constant 2011 international $)	$\gamma_{GDP/Cap}$
2010	9,230	
2011	10,041	**8.8%**
2012	10,756	**7.1%**
2013	11,525	**7.2%**

Data from World Bank Database, for the year 2013.

Increases in a country's GDP provide a measure of how quickly economic activity is increasing and thus how quickly a country is developing. China's GDP has grown at a rate of nearly 10% per year for many years now, and is considered to be developing very quickly. During the same period, Cote D'Ivoire has been economically stagnant, with a real GDP growth rate in the range of 1-2%.

GOING FURTHER

Exponential Growth

When the growth rate, γ_X, is equal to a constant r, we obtain:

$$\frac{X_{t+1} - X_t}{X_t} = r \leftrightarrow X_{t+1} = (1+r)X_t \ , \forall t$$

In other words, the value at $t+1$ is proportional to the value at t. We further have that:

$$X_{t+2} = (1+r)^2 X_t \ , \forall t$$

$$\ldots$$

$$\boxed{X_{t+n} = (1+r)^n X_t \ , \forall t} \quad (*)$$

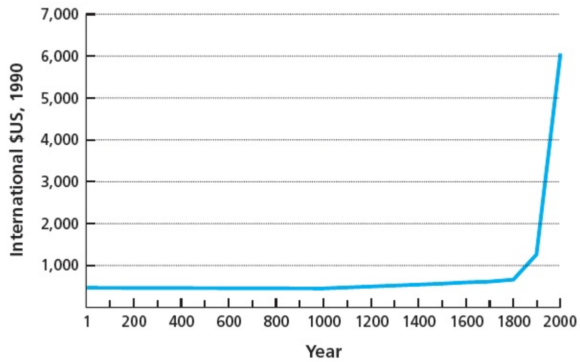
As a result, with a constant growth rate, we have exponential growth. Not convinced that expression $(*)$ above is exponential growth? To make it more apparent, let's use the following math formula: $a^b = e^{\ln(a^b)} = e^{b.\ln(a)}$.

We can rewrite expression $(*)$ as: $X_{t+n} = e^{n.\ln(1+r)} X_t \ , \forall t$

Substituting R for $\ln(1+r)$, we get:

$$\boxed{X_{t+n} = e^{nR} X_t \ , \forall t}$$

Exponential growth is commonly observed in nature. For example, it describes how fast a population increases with no resource constraint, or how fast a contagious disease spreads when there is no immunization available. We also observe exponential growth in economics. Figure 1.2, 1.3, and 1.4 in the main book illustrate what exponential economic growth looks like. The exponential growth curve has a very specific shape: it begins with low levels, slowly increases for a while, and then suddenly increases dramatically. For this reason, it has been dubbed the 'Hockey Stick' curve.



1.2 Gross world product per capita (1990 International Dollars)

Source: Bolt, J., and J. L. van Zanden. 2013. "The First Update of the Maddison Project: Re-Estimating Growth Before 1820." Maddison Project Working Paper 4.

You can calculate growth rates using either the discrete or continuous formula:

- Discrete formula: $(1+g)^T = \frac{y_t}{y_0} \Leftrightarrow g = \left(\frac{y_t}{y_0}\right)^{\frac{1}{T}} - 1$
- Continuous formula: $e^{rT} = \frac{y_t}{y_0} \Leftrightarrow r = \frac{1}{T}\ln\left(\frac{y_t}{y_0}\right)$

EXAMPLE

World per capita income in 1820: $665
World per capita income in 2008: $7,603

Discrete formula: $g = \left(\frac{7{,}603}{665}\right)^{\frac{1}{188}} - 1 = .0131 = 1.31\%/\text{year}$

Continuous formula: $r = \frac{1}{188}\ln\left(\frac{7{,}603}{665}\right) = .0130 = 1.30\%/\text{year}$

D. RULE OF 70

Suppose you know that country A has been growing at an average rate of 5% for the past year. You wonder how long it would take for that country to double in GDP. The Rule of 70 provides you with a very easy way to figure this out. It states that the number of years necessary for GDP to double can be approximated by 70 divided by the exponential growth rate, r, in percentage. This rule is applicable to any quantity that is growing, such as a city with a growing population.

DEFINITION: Rule of 70

If N is the number of years it takes for the quantity to double, and r is the rate of growth, then:

$$N \approx \frac{70}{r}$$

EXAMPLE

If a country has a constant GDP growth rate of 7%, its GDP will double within 70/7 = 10 years. If a country has a constant GDP growth rate of 10%, its GDP will double within 70/10 = 7 years.

GOING FURTHER

Where is the rule of 70 coming from?

Suppose we are studying a particular quantity X(t) that is growing over time at a constant rate r. We have that:

$$X(t+2) = (1+r)^2 X(t)$$

$$\dots$$

$$X(t+N) = (1+r)^N X(t) \quad \Leftrightarrow \quad \frac{X(t+N)}{X(t)} = (1+r)^N = e^{N.\ln(1+r)}$$

Since $\log(1+r) \approx r$ when r is small, we get: $\frac{X(t+N)}{X(t)} \approx e^{Nr}$

The rule of 70 consists in solving for N such that $\frac{X(t+N)}{X(t)} = 2$,

i.e. such that the quantity of interest X doubles.

$$e^{Nr} = 2 \Leftrightarrow Nr = \ln(2) = 0.6931 \ldots \approx 0.70$$

As a result, the number of years N necessary for X to double is equal to:

$$\boxed{N = \frac{0.70}{r} = \frac{70}{r\ (in\ percent)}}$$

E. THE LOG SCALE

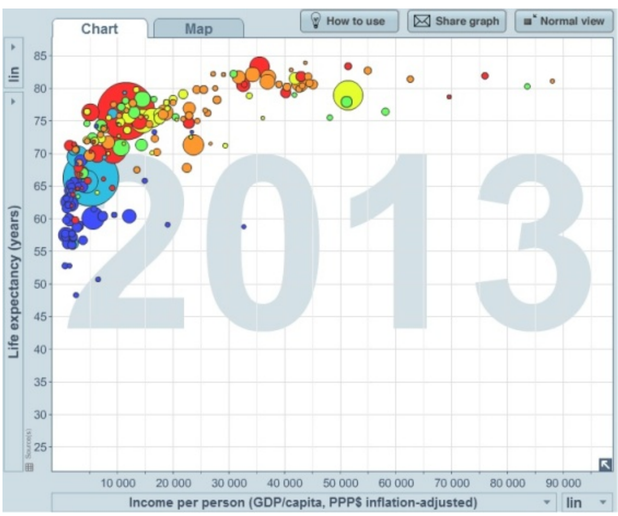
DEFINITION: LOG scale The logarithmic scale is a nonlinear scale that is very useful when dealing with values that have a large range (e.g. in case of exponential growth).

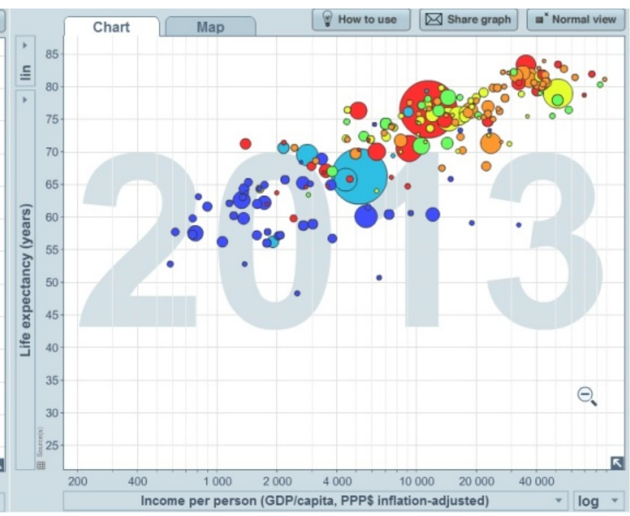
A linear scale displays data where the interval between each increment is always the same: e.g. 1 -> 2 -> 3 -> ... A logarithmic scale displays data where the interval is a multiple of a specific number: e.g. 1 -> 10 -> 100 ->...

EXAMPLE Using Gapminder, we plotted Life expectancy vs. Income per person. On the left-hand side, we use a linear scale for the x-axis; on the right-hand side, we use a log scale.

We note that, on the left-hand side, points corresponding to the lowest values of income per capita are all very packed together: this makes the graph hard to read. On the right-hand side, these values are now well-dispersed, and it becomes easy to distinguish a country at $1,000 income per capita from one at $2,000.

In addition, we note that on the left-hand side, points corresponding to the highest values of income per capita are very dispersed. These values are easy to read but the space on the graph is not well used: the right part of the graph is almost empty. When switching to log scale, these points become much closer to each other, and our plot becomes more graphically appealing.





GOING FURTHER

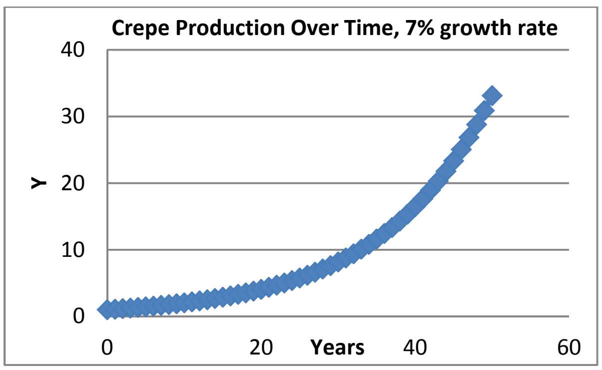
Log scale and rate of growth

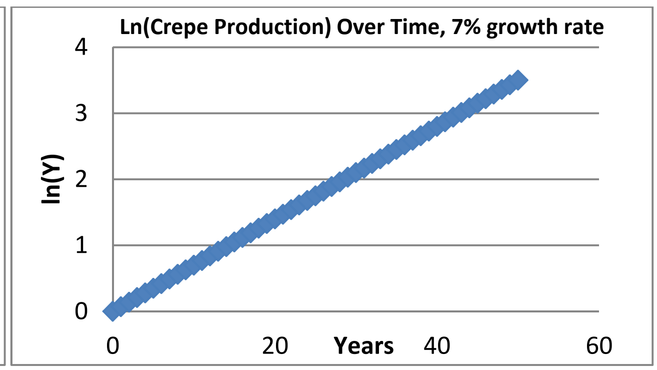
Another convenient feature of using the logarithm of values when plotting quantities is that it graphically reveals the rate of growth of that quantity.

Indeed, we simulate below a fictitious economy producing crepes. Throughout time, the economy is able to produce more and more crepes at a growth rate of 7%. This yields the exponential curve on the left-hand side below. When plotting the logarithm of the values, the curve is transformed into a line whose slope is the exact exponential growth rate.

Not convinced? Look at the point on the right-hand side graph where years equal 30. This corresponds to about ln(Y) = 2.

The slope is therefore equal to 2 / 30, about 0.07, or 7%. Exactly the growth rate!





ACTIVITIES

EASY

A. The rule of 70

1) If an economy grows at 3.5% a year, approximately how long will it take for it to double?
2) Suppose a country grew on average at around 1.8% from 1800 to 2000. How many times bigger is the economy in 2000?
3) An economy experiences economic growth, but no change in population. In the year 2014, per capita income is $15,000. At 2% average annual growth, when will this country reach $60,000 per capita?

Answer: 1) 20 years; 2) 70/1.8 = 39 years to double i.e. about 40 years, so in 200 years, it will double 5 times: 2^5 = 32. The economy will be 32 times bigger; 3) 70 years

MEDIUM

B. The rule of 70

Based on the explanations of where the rule of 70 comes from:

1) What would be a rule of thumb for a quadrupling time?
2) What would be a rule of thumb for trebling and quintupling times?

Answer: 1) Quadrupling = doubling of doubling, i.e. rule-of-140 (because ln(2x2)=ln(2^2)=2ln(2) = 2x70=140); 2) Trebling: rule-of-110 (because ln(3)=1.099) and quintupling: rule-of-160 (because ln(5)=1.6).