Modeling Companion D
Solow Model

LEARNING OBJECTIVES

- Understand the basics of the Solow model

MOTIVATION

Chapters 1 to 5 have shown that there exist vast inequalities of wealth across countries: some countries developed while others stayed poor. Economists have attempted to explain why economic growth would occur in some places and not in others. In 1987, Robert Solow was awarded the Nobel Prize for his contributions to the theory of economic growth. One of his key contributions was the Solow model, which to this day remains a building block of modern growth theory.

The Solow model provides a simple but useful framework of how economic growth is generated through capital accumulation. In the next companion, we will augment the Solow model by modifying some assumptions, which will lead to the creation of poverty traps. Studying these models is a useful way to think through various factors that could explain why some countries developed while others stayed poor.

In this companion, we will present the basics of the Solow model and introduce a modeling activity using the excel document "Modeling companion D Solow Model – Excel." We will only explain the basics of the model. For a more detailed presentation of the Solow model, the reader should refer to an undergraduate level macroeconomics textbook such as "Macroeconomics" by Charles Jones, or for a more advanced description of the model, refer to "Economic Growth" by Robert J. Barro and Xavier I. Sala-i-Martin.

THE SOLOW MODEL

<u>Introduction</u>

The Solow model is an attempt to answer the following question: what leads to low levels and low growth rates of income? The model interprets growth as a process of factor accumulation: in other words, income per person, y, comes from accumulating capital per person, k. As a result, a proximate answer for the question above is the following:

A country has low levels and low growth rates of income because:

– the country has not accumulated crucial factors (H, K)

– the country is not combining factors effectively (A)

<u>The two fundamental equations of the Solow model:</u>

1) The Solow model uses a neoclassical production function: a function with constant returns to scale (CRTS) and diminishing returns to one input.

The Cobb-Douglass is an example of such a function: $Y = A\,K^{\alpha}\,L^{1-\alpha}$ with $0 < \alpha < 1$. Here K stands for capital, L for labor, A for technology, and Y for income.
This function can be re-written as follows:

$$y = \frac{Y}{L} = \frac{A\,K^{\alpha}\,L^{1-\alpha}}{L^{\alpha} * L^{1-\alpha}} = A\left(\frac{K}{L}\right)^{\alpha} = Ak^{\alpha}$$

where k is capital per worker and y is income per worker.

2) One key aspect of the Solow model is the way capital accumulates. The Law of Capital Accumulation, or the Law of Motion, can be written as follows:

$$k_{t+1} = k_t + sy_t - (n + \delta)k_t$$

where k_t is capital per worker at time t, s is the saving rate, y is the income per capita, n is the population growth rate, and δ is the depreciation rate.
This equation tells us how capital per worker (k) changes over time.

– How does k go up?
    → By saving and investing a proportion of output ("sy")
– What forces push k down?
    → Tools, machines, and other capital goods depreciate and need replacement ("δk")
    → Population is growing ("nk") and these new workers need to be equipped with k

The Law of Motion can be rewritten as:

$$\Delta k = k_{t+1} - k_t = sy_t - (n + \delta)k_t$$

<u>The dynamics of the Solow model:</u>

The diagram below summarizes the dynamics of the Solow model. The Law of Motion tells us that when the amount of savings is greater than what disappears due to depreciation and population growth, capital per worker will grow. When the amount of savings is smaller than the amount needed for replacement, capital per worker decreases.

$$\Delta k = sy_t - (n + \delta)k_t$$

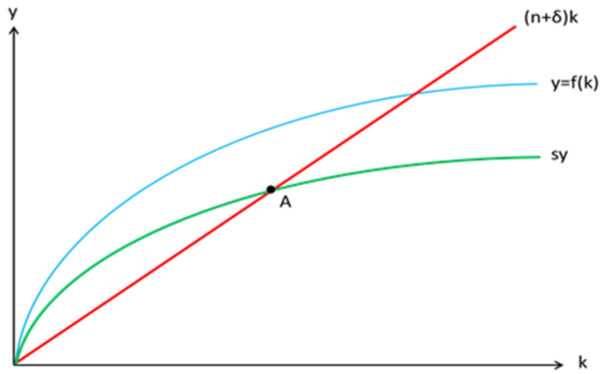
If $\underbrace{sy_t}_{\text{savings}} > \underbrace{(n + \delta)k_t}_{\text{replacement}} \Longrightarrow \Delta k > 0$

Capital per worker grows, Ie accumulates

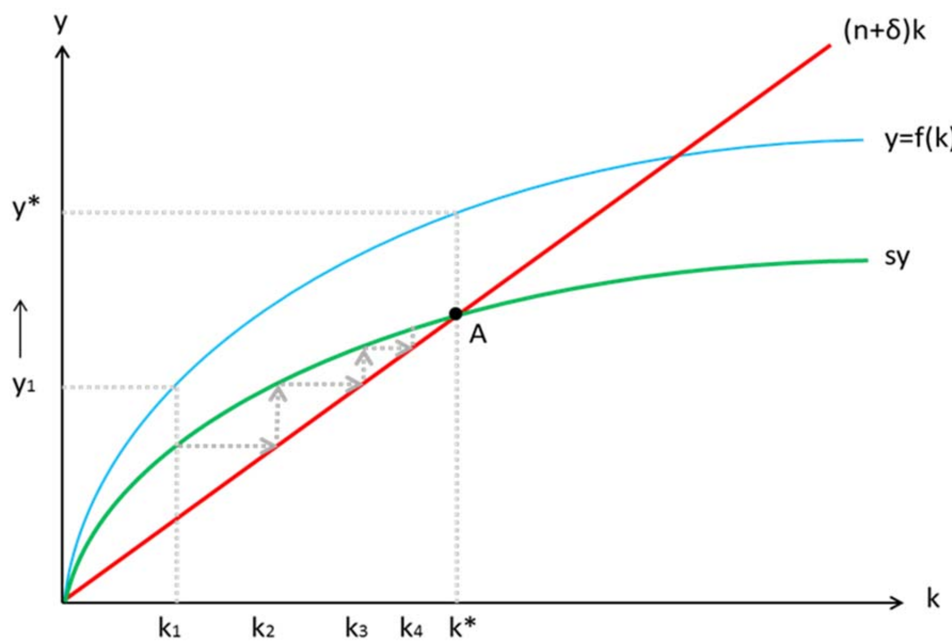
If $\underbrace{sy_t}_{\text{savings}} < \underbrace{(n + \delta)k_t}_{\text{replacement}} \Longrightarrow \Delta k < 0$

Capital per worker decreases

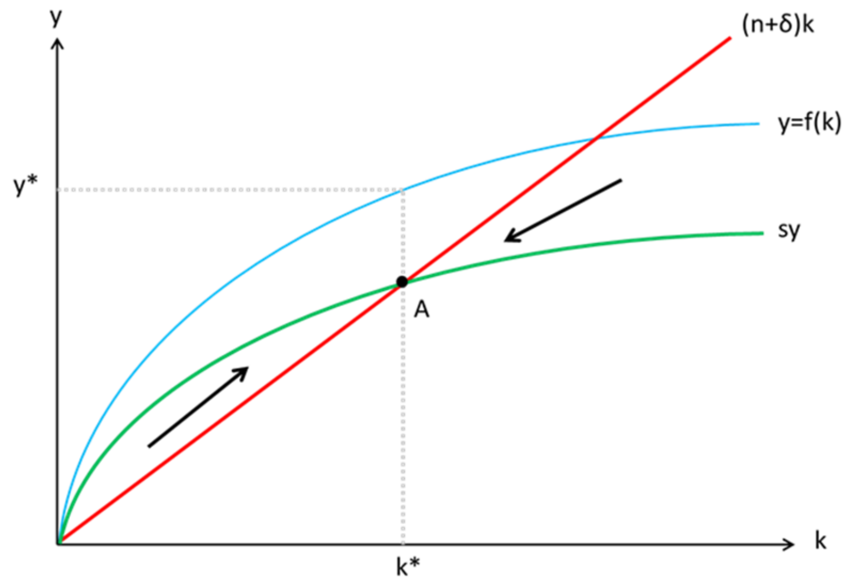
The Solow model on a graph:
On the graph below, the country's production function (y = f(k)) is plotted in blue. The amount savings (sy) is represented by the green curve, while the amount necessary for replacement is represented by the red line.
Point A represents the equilibrium of the system: at that point, the amount saved exactly equals the quantity necessary for replacement such that capital per worker remains constant. At point A, $\Delta k_t = 0$.



The dynamics of the Solow model on a graph:
What if a country starts with low capital per person? Say a country starts with k_1. At k_1, we note that the green curve is above the red line. This means that savings are greater than replacement. As a result, capital per worker will increase, and we will move to the right on the x-axis to reach k_2. At k_2, savings are still greater than the amount necessary for replacement, as a result capital per worker grows and we reach k_3, etc., until we reach k^*. You can do the same exercise supposing a country starts with a value of capital per worker greater than k^*. Eventually capital per worker will converge down to k^*.

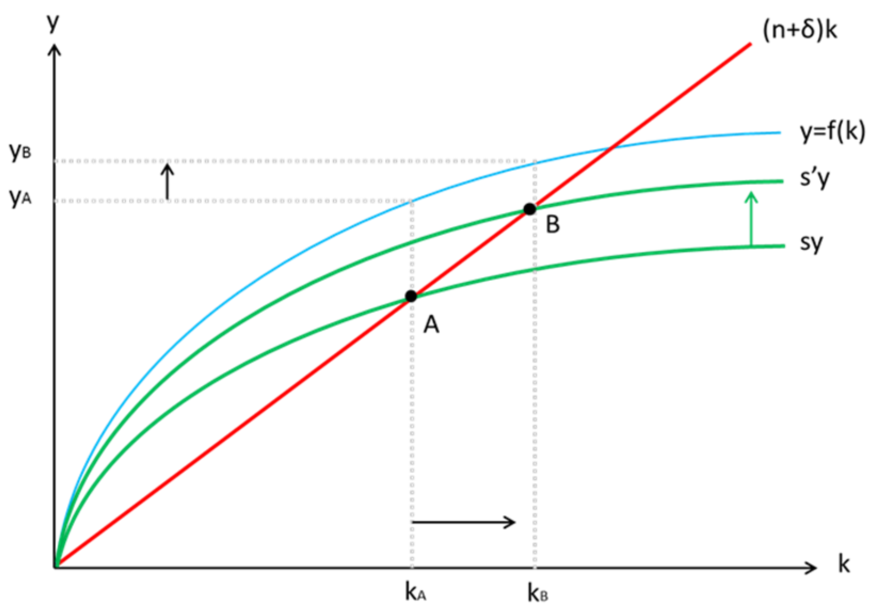


Conclusion: no matter what capital per person we start with, income will always converge to the equilibrium determined by s, n, and δ.

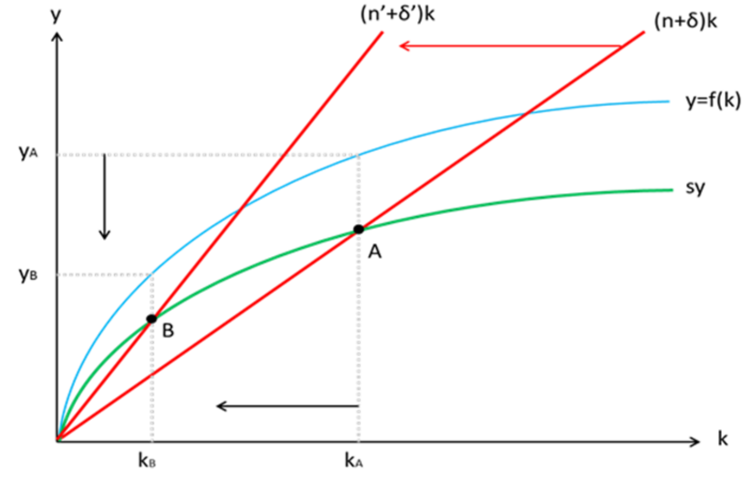


What happens in the Solow model when the saving rate increases?

As the graph below illustrates, the equilibrium level of income per person increases.



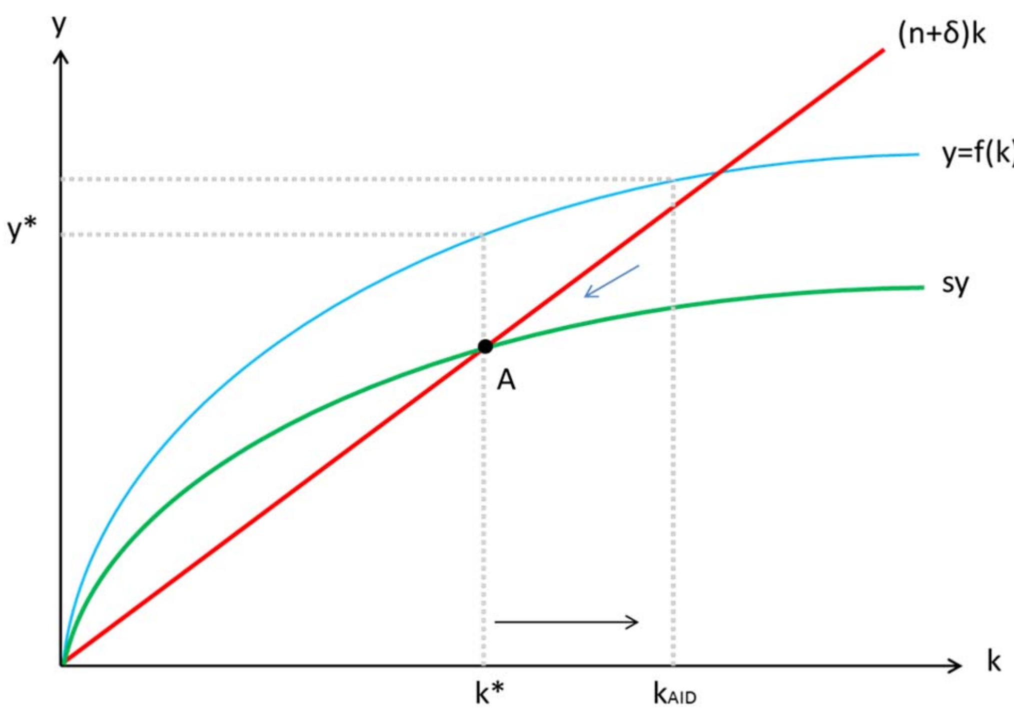
What happens in the Solow model when the depreciation rate or population growth rate increases?



As the graph above illustrates, the equilibrium level of income per person decreases.

What happens in the Solow model when a country receives foreign aid in the form of extra capital?

As the graph below illustrates, income per worker will temporarily increase, but eventually the dynamics are such that capital per worker will converge back to the equilibrium level of income per person y^*.



The conclusions of the Solow model so far:

- Growth in y occurs only as countries move toward the steady state.

At the ***steady state***:

- There is no growth in income per capita, y.
  (because we're saving just enough to replace needed capital)
- Income, Y, does grow though, at rate n.
  (because y = Y/L, and L increases at rate n)
- Countries with higher saving rates and lower population growth rates should have higher levels of income per capita.

- Countries with low k should grow more quickly than countries with k closer to the steady state
  → This predicts high growth in poor countries, conditional s, n, and δ.
  This is called **CONDITIONAL CONVERGENCE**: if countries have the same saving rate, population growth rate, and depreciation rate, their per capita income levels should converge, in the long run, to the same level.

Do we observe these trends empirically? In rich countries, growth does not seem to be moving toward zero. It even seems that in some cases, growth may be faster in rich countries than in poor countries (DIVERGENCE).

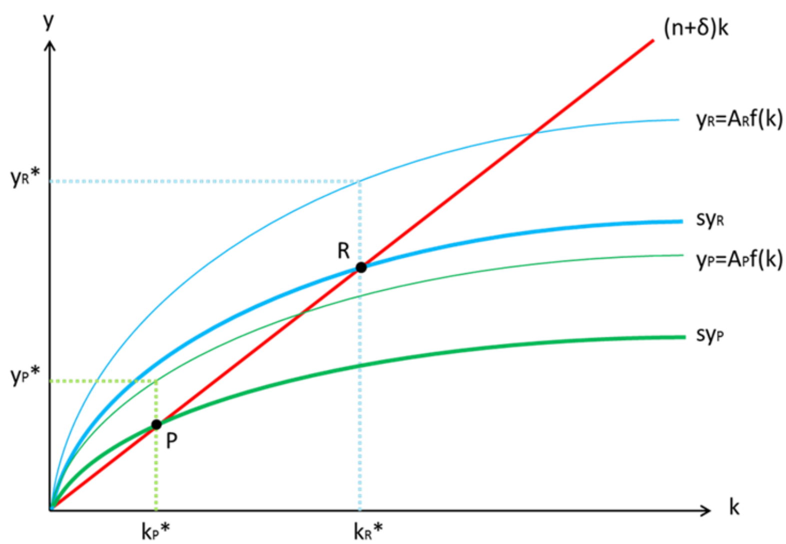
Technological innovation in the Solow model

We previously wrote the production function of the country as a simple function of capital per worker: $y = f(k)$. Let's augment this production function. Let's say that the production function of a country is a function of both capital per worker and technology: $y = Af(k)$.

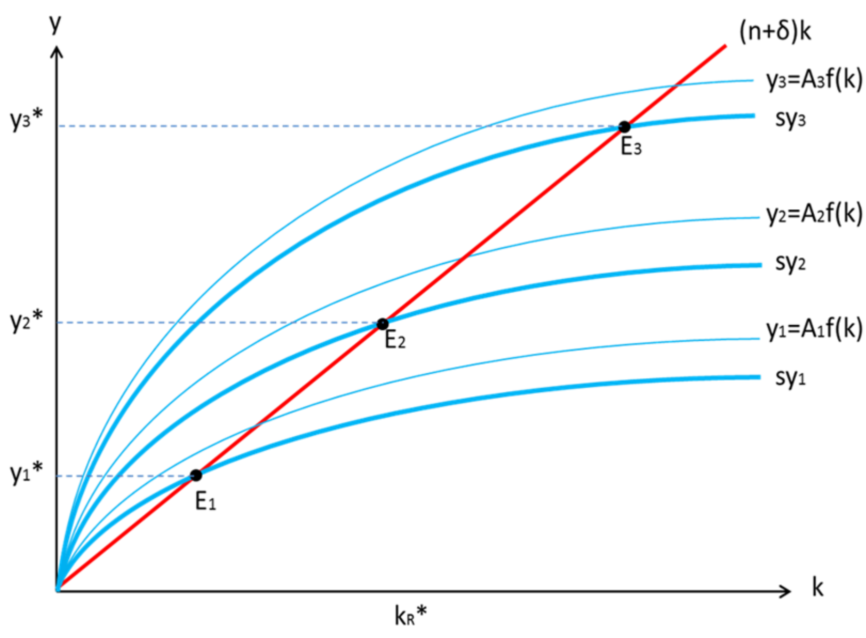
By writing "A", we do not explicitly model technological innovation; in other words the value of A is determined by the model—we say that A is exogenous.

A is also referred to as total factor productivity (TFP). Empirically, it corresponds to the growth we observe but for which L and K cannot account for. We interpret TFP broadly as technology.

We should then ask what happens if countries have different levels of technology. The graph below illustrates the case of a country with a high level of technology (A_R) and a country with a low level of technology (A_P). The different levels of technology imply different equilibrium levels of income per capita. The country with $A_R$ends up with a high level of income per capita y_R^* while the country with $A_P$ends up with a low level of income per capita y_P^*. Hence, we can conclude that more technology induces higher levels of income per capita.



What if technology grows at rate g? The graph below illustrates what happens when the level of technology increases from $A_1$to A_2 to A_3. We see that as technology increases, the equilibrium will keep going up from $E_1$to E_2 to E_3; similarly, the equilibrium levels of income per capita keep increasing from y_1^* to y_2^* to y_3^*. As a result, technological growth translates into income growth!



<u>The final conclusions of the Solow model:</u>

- In the long run, k and y grow at the rate of technological growth
  → If there is no technological growth, there is no income per capita growth.
- Differences in the long run growth rates, and instances of convergence and divergence are all driven by differences in technological growth.

MODELING ACTIVITIES

Suppose we live in a Solow-type economy. This means:

- the economy produces output according to the following production function:
  $y_t = A\, k_t^{\alpha}$ (1)
  where $0 < \alpha < 1$, y is the output per worker, A is the level of technology and k is capital per worker.
- Capital accumulates according to the following 'Law of Motion':
  $k_{t+1} = k_t + sy_t - nk_t$ (2)
  where k is capital per worker, s is the saving rate, and n is the population growth rate.

The t subscript denotes the time period (e.g. if t = 2000 then t+1 = 2001, and so on).

Open the spreadsheet 'Modeling Companion D Solow Model.xls'.

Question 1

In tab "Solow 1," we have simulated a Solow economy with the following parameters: s = 0.2; n = 0.04; alpha = 0.33 and A = 1. We take ko = 1. On graphs 1 and 2, we plotted income per capita with time and the growth rate of income per capita with time.

a) What happens if we change the initial condition of k from 1 to 50? Why?

b) Take ko = 1 again.
What happens if you change the saving rate from 0.2 to 0.6? Why?
You do not have to provide a graph. Just describe qualitatively what happens.

c) What happens if you change the population growth rate from 0.04 to 0.1? Why? (Use s = 0.2 and ko = 1).
You do not have to provide a graph. Just describe qualitatively what happens.

Question 2

In tab "Solow 2", we have simulated the same Solow economy as in question 1. We will now simulate what happens when the economy receives external transfers such as foreign aid. Say the economy benefits from a transfer of $2 in 2010 in capital per worker. To simulate this scenario in your excel spreadsheet, manually add '+2' in the cell where k(t=2010) is located.

a) Plot on the same graph:
- the evolution of income per capita, y, with time when the economy benefits from the transfer as explained above.
- the evolution of income per capita, y, with time when the economy does not get any transfer (i.e. what is simulated in question 1)).
b) What happens to income per worker?
c) What happens to the growth rate in the short-term and in the long-term?

d) Does the transfer change any of the dynamics of the Solow model?
e) From this simulation, what can you conclude about the role of foreign aid in a Solow-type economy?

<u>Question 3</u>

In tab "Solow 3", we have simulated what happens when we add technological growth to the Solow model. In graph 1 and 2, we have plotted income per capita and growth of income per capita for "Economy 1" (the economy simulated in the tab "Solow 1") and for "Economy 3" (the economy simulated in the tab "Solow 3").

a) Change the growth rate of technology in cell D6 from 0 to 0.01. Do Economy 1 and Economy 3 have the same long-term dynamics? Is there a steady state y*? If so, what is it? How do the long-term growth rates compare?
b) From this simulation, what can you conclude about the role of technological innovation?